SEMI-ANNUAL PROGRESS REPORT

GRANT NGR-44-005-039

ADVANCEMENT OF THE GENERAL THEORY OF MULTIPLEXING

by

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A. Introduction

1. Background of Grant

The University of Houston was awarded a research grant by the National Aeronautics and Space Administration entitled: "Advancement of the General Theory of Multiplexing with Application to Space Communications." The research grant (NGR 44-005-039) was for the sum of $51,356.00, for a period of approximately one year beginning June 1, 1966.

The grant was for the purpose of supporting research on the theory of multiplexing in order to develop new multiplexing systems for use in communications with spacecraft. Multiplexing is the process by which several signals are combined to be sent over a single wire or radio channel.

The work is being performed by members of a Communication Research group in the Department of Electrical Engineering. Technical supervision is being provided by the Information Systems Division of the Manned Spacecraft Center of Houston.

2. Organization of Report

This report covers the technical and administrative details of the grant. The technical objectives, preliminary results, and plans for further investigations are discussed in detail. Reference to the table of contents is suggested for a complete breakdown of topics.

B. Technical Progress

1. Technical Objectives

The purpose of this investigation is to conduct fundamental research leading to an improved general theory of multiplexing. New types of multiplexing methods will be developed, and their performance compared with presently known methods. Emphasis will be placed on the techniques most applicable to space communications in general, and unified deep space systems, such as the Apollo S-Band System, in particular. Improved mathematical methods of multiplexing system design will also be studied.

2. Preliminary Results

a) The General Theory of Multiplexing

The general theory of conventional multiplexing systems has been reviewed. It has been determined that conventional multiplexing systems are members of the orthomux class. All the literature extant on new multiplexing systems has been reviewed.

It has been determined that the optimum multiplexing system depends on the assumptions made about the channel. For a simple baseband channel which consists only of additive white, Gaussian noise, it has been proven that all orthomux systems perform equally well. This rather remarkable result is modified, however, for more complicated channels.

For a channel which band limits the frequency spectrum of the transmitted signals, it turns out that the optimum orthogonal signals for the orthomux system are the prolate spheroidal wave functions. Since these wave forms are difficult to generate, consideration is being given to generation of approximating functions. The performance of orthomux systems based on these approximating functions will be compared to that of the optimum system.

If the channel has a peak power limitation (as do all practical transmitters) then a different result is obtained. The optimum multiplexing system is that which uses binary (digital) waveforms. These waveforms have a peak-to-average power ratio of one which is clearly optimum. Consideration will be given to which of the types of binary combination methods will lead to the simplest implementation, since all binary codes will perform equally well in terms of peak-to-average ratio.

After those results have all been obtained, consideration will be given to the application of simultaneous constraints: simplicity, bandwidth minimization, and peak-to-average power minimization. This study will produce the most realistic optimal system for space communications design.

It is planned to present the key results of this task at the National Telemetry Conference in San Francisco in May.

b) Orthogonal Multiplexing Based on Easily Generated Functions

All orthomux systems perform equally well for a channel which consists only of additive white Gaussian noise. This channel corresponds to transmission at baseband (over a wire line perhaps) with large bandwidth capability. Thus, the only criterion for optimization for such a channel is that of equipment simplicity.

Several of the more promising sets of orthogonal functions have been investigated. In addition, general techniques have been developed to improve the characteristics of these functions. To illustrate this, consider a particular set of orthogonal functions $\{0_n(t)\}$. The set of functions can be modified as the following integral shows:

$$\int_0^T \left(0_n(t)\ p(t)\right)\left(0_m(t)\ P^{-1}(t)\right) dt = \delta_{nm}$$

The P(t) function does not affect the characteristic of orthogonality, but selection of the proper P(t) can significantly reduce the bandwidth of the transmitted signal, make the transmitted signal easier to generate, etc. Also the channel signals can have a different function of time or a constant added to each one. It is very helpful in reducing bandwidth and peak-to-average power requirements to shift the level of a signal (add a constant) and then to invert alternate pulses in the transmitter. The receiver can reverse this or any other fixed process.

Constraints can be placed on the characteristics of a set of orthogonal functions prior to the generation of the functions. The Gram-Schmidt procedure is used to generate orthogonal sets of functions. The peak-to-average power ratio can be limited prior to application of this procedure, for example. It is thus possible to synthesize a set of constrained orthogonal functions and thus produce orthomux systems with desired characteristics.

An example of a very promising set of orthogonal functions is the real exponential set, which is given by

$$O_n(t) = \sum_{k=1}^{n} C_{nk}\, e^{-kpt}$$

These functions are very easy to generate, being the output of RC networks with an impulse applied to the input. The interval of orthogonality is from t=0 to $t=\infty$. By choosing P so that the exponentials die out at a desired rate, the functions can be truncated at some finite time t=T with negligible crosstalk. An orthomux system based on these functions has been investigated in detail. The results of this investigation will be presented to the Communications Technology Group of the 1967 IEEE Convention in New York.

c) Analog and Digital Simulations of Multiplex Systems

(1) Analog Simulation

An Applied Dynamics Analog Computer is being used to simulate a general three-channel orthomux system. The analog computer setup does not change during the course of the simulation as various system parameters are changed, because the various operations such as multiplication, integration, and sample-and-hold are necessary and identical despite system parameter changes. Therefore it is possible to measure system performance under varying conditions of noise, bandwidth, etc. with a minimum of effort. For this reason the analog computer is ideally suited for orthomux system simulation.

At present the real exponential set of orthogonal functions forms the signal set which is being simulated. The effects of time and frequency domain truncation on crosstalk, the effects of various kinds of noise, and the effects on performance of certain improvement techniques will be determined. Many of the interesting performance parameters have already been calculated for the real exponential orthomux system, and these calculated values will be compared with measured values from the computer simulation.

Frequency truncation and peak waveform clipping are examples of occurrences which produce crosstalk in multiplex systems. These events are very difficult to describe by an exact mathematical representation but can be handled easily by the computer.

To completely change the set of orthogonal functions under investigation is also a simple task and again this does not disturb the basic computer setup. It is only necessary to change the function generators. Thus several of the more promising sets of functions will be investigated when the real exponential set simulation is concluded.

(2) Digital Computer Simulation

The general orthogonal multiplexing system is also being simulated in the time domain by use of a digital computer program. This program will perform all operations required by an orthomux system in discrete steps, utilizing a general orthogonal function set, ten general messages, and a variable integration interval. As an output it will calculate the appropriate channel messages. Provisions for varying the input for any or all channels is being included. The program will include as subroutines methods for generating the required orthogonal functions and methods of calculating crosstalk.

When suitable results have been obtained for the message held constant over the period of integration, additional complexities will be added, such as variations in the message during the time of integration, the addition of sine wave interference, and the addition of peak limiting.

The results of the simulation programs will be presented at the IEEE International Communications Conference in Minneapolis, Minnesota in June 1967.

d) Design of A Conventional Multiplex System for the Apollo Applications Program

(1) Design Philosophy and Constraints

Investigation of the probable course of the Apollo Applications Program indicated that the most likely mission for which a complete communications system redesign is necessary is the manned Mars landing mission.

Studies of equipment capabilities at the time of this mission, which will be in 1973, indicated that it was reasonable to expect the equipment performance shown in Table 1 . These estimates are considered very conservative and in all instances are based upon conventional devices which exist at the present time and merely represent extensions of the present state of the art. It is hoped and expected that several breaks-through in techniques would make certain unconventional devices available at the time of the mission; however, it is not feasable from a communications point of view to consider them in a communications system design.

Although it is difficult to pinpoint exact mission requirements at this time, it seems probable that an absolute minimum requirement is for a continuous ranging and tracking, voice, and two-way telemetry capability to a distance of 50 million nautical miles. For this reason effort was devoted toward finding the optimum way to combine these functions, subject to the equipment restraints of Table 1.

If the constraint is added that any system chosen utilize current techniques and be compatable with present ground stations, then the multiplexing scheme chosen is of necessity frequency division multiplexing.

TABLE I

PREDICTED SYSTEMS CHARACTERISTICS

Spacecraft to Earth

	Present	1973
Frequency	2.2 GHz	2.2 GHz
Transmitter Power	20 watts	200 watts
Transmitter Antenna Gain	28.6 db	38 db
Receiver Antenna Gain	52 db	60 db
Receiver System Temp.	270°K	25°K

Earth to Spacecraft

	Present	1973
Frequency	2.2 GHz	2.2 GHz
Transmitter Power	10 kw	100 kw
Transmitter Antenna Gain	52 db	60 db
Receiver Antenna Gain	28.6 db	38 db
Receiver System Temp.	4600°K	730°K

The modulation chosen must be narrow band because of the pseudo noise ranging codes. As a result of the above, the problem for this case reduces to optimizing such parameters as subcarrier frequency, modulation index, and bandwidth. This problem is described in more detail in Section 2.

If the above mentioned constraint of compatability with present ground stations is relaxed to allow for the integration of new systems on a small scale, then it appears that the optimum method to transmit the required information is to form one composite bit stream and use it to phase-shift-key the carrier. The advantages of this system are more fully described in Section 3.

(2) Optimization of a Narrow Band PM FDM Signal

It is present practice to rate the performance of a single RF channel by the range equation expressed in decibel notation. This is:

$$\begin{aligned} SNR = {} & P_M + G_M + G_R - 10 \log_{10} (KBT) \\ & -20 \log_{10} f - 20 \log_{10} R + 37.8 \end{aligned} \qquad (1)$$

where

P_M = Transmitted power

G_M = Transmitting antenna gain

G_R = Receiving antenna gain

K = Boltzman's constant

B = System bandwidth

T = Effective system noise temperature

R = Range in nautical miles

f = Frequency in MH_Z

This figure is then compared with the SNR required for satisfactory performance to obtain a quantity commonly referred to as performance margin M, where

$$M = SNR_{actual} - SNR_{required} \qquad (2)$$

This procedure can be extended to a multiplexing system by considering the multiplexing process as a scheme which allots certain fractions of the available power to each channel. This division of the available power can be taken into account in Equation (1) by the addition of $10 \log_{10} P$, where P is the fraction of power allocated to each channel. The resulting signal-to-noise ratio can then be calculated for each channel of the system and compared with the required signal-to-noise ratio to obtain the margin for each channel.

The equations for each channel can then be grouped together to form a system of equations which completely characterize the system.

$$\begin{array}{ccccccccc}
A & - & B_1 & + 10 & \log_{10} P_2 & - & (S/N)_{REQ1} & = & M_2 \\
A & - & B_2 & + 10 & \log_{10} P_2 & - & (S/N)_{REQ2} & = & M_2 \\
\cdot & & \cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot & & \cdot \\
A & - & B_n & + 10 & \log_{10} P_n & - & (S/N)_{REQn} & = & M_n
\end{array}$$

where A = Sum of losses and gains common to all channels

B_n= Sum of losses and gains common to nth channel

P_n= Fraction of energy in the nth channel

$(S/N)_{REQn}$ = Required signal-to-noise ratio in the nth channel

M_n = Performance margin of the nth channel.

For the system of interest P_n has been shown to be of the form

$$P_n = \cos^2 (B_o)\, J_o^{\,2} (B_1)\, J_o^{\,2} (B_2) \ldots J_1^{\,2} (B_n) \ldots . J_o^{\,2}(B_n)$$

where B_n is the modulation index of the nth channel.

The optimization criterion chosen for the above system of equations is that the variables B_n and P_n should be chosen so that the set of Ms are equal valued and as large as possible for a given range. If such a criterion is chosen, the system will fail under the most extreme conditions and all channels will fail simultaneously. If all channels did not fail together, there would be no reason to multiplex the signals, but, instead, one should merely discard the failed channel and combine the remaining channels in an optimum manner.

A computer search routine was developed to pick the optimum values of modulation index based upon the above criterion. The form of P_n and the equipment characteristics are left as input variables to be specified at the time of execution. The program is written in the MAD language for the IBM7094 but can be modified for use with other scientific languages. The program specifically chooses the values of modulation index which best satisfy the optimization criterion and then calculates the performance margins for these indices. The program can also optimize parameters such as bandwidth, bit rate, and operating frequency.

Preliminary performance margins have been obtained by this method for a number of modes of operation. Further effort is being devoted to refinement of the program and determination of the maximum telemetry bit rate.

3. <u>A Spacecraft to Ground Digital Link</u>

For a channel which is peak power limited but not bandwidth limited, the proven optimum means of communications is simplex signaling . In the binary case this theoretical optimum is most nearly approached in practical systems by phase shift keying, with coherent demodulation and matched filter detection.

With the optimum choice of modulation index, bandwidth, and carrier frequency, it was determined that at Martian distances approximately 3K bits of information could be transmitted from a spacecraft to Earth using the conventional techniques of Section B, but with digital signaling a possibility of transmitting approximately 18 KBPS exists.

The main drawbacks to such a system at the present time are synchronization problems, carrier tracking problems, and the problem of combining the pseudo random ranging code with the other PCM data. At the present time considerable effort is being devoted to the discovery of techniques to alleviate these problems, to the integrations of the techniques into the system, and to the analytic and experimental analysis of these techniques.

e) Use of Analytic Signals for Analysis of Multiplex Signals

The theory of analytic functions is used as a new approach to the study of conventional multiplexing schemes. The multiplexing process involves multiple modulation techniques. The most general form of modulated waves exhibit simultaneous phase and envelope fluctuation.

Phase and envelope relationships described in mathematical formulation have a strong dependence on the real and complex zeroes of the actual wave. The properties of analytic signals allow the formulation of a general theory of zero-representation of the original signal that contains its informational attributes. The mathematical representation of zeroes of a periodic real signal has physical significance when related to the spectra, phase, and envelope fluctuations. The zero-pattern representation offers the advantage of showing the phase envelope fluctuation in the time domain, and a frequency domain representation can be based on the frequency spectrum given by the Fourier series coefficients.

The zero representation is used to study different forms of modulated waves based on the principle of factorization in terms of the Fourier series expansion and on the property of zero-pattern superposition resulting from the product of two signals. Multiplicative processes are the most amenable to a zero-based description, and special attention is devoted to the study of the characteristics of this type of modulated wave from its zero pattern configuration. The zero manipulation property makes mathematical representation simpler.

A qualitative comparison between different multiplexing systems is also possible in terms of the zero patterns. Tensor analysis is employed in the multi-space in which the zeroes of multiplexed signals are located. The evaluation of crosstalk in terms of tensor forms results in simplification when automatic computing techniques are used.

A complete computer program has been developed in two different versions to yield the "zero-loci" of the modulated waveforms. Some guidelines to compute the inter-channel crosstalk were also developed.

The above topics are treated in detail in a masters thesis by Louis C. Puigjaner entitled "Analytic Signals and Zero-Locus in Multiplexing Systems," August 1966, The University of Houston. The thesis also contains a comparison of different multiplexing systems.

3. Technical Plans

The technical plans for the remaining six months of the grant are outlined below. The technical plans for the proposed renewal of the grant are detailed in the attached proposal.

From December 1966 to June 1967 the following tasks will be performed:

a) General Theory of Multiplexing

The optimum types of multiplexing systems have been determined for certain simple channels. There remains to determine approximating functions for the prolate spheroidal functions for the bandlimiting channel and the optimum type of digital code for the peak power limited channel. Then the optimum type of multiplexing will be considered for more realistic channels. It is planned to present the key results of this task at the National Telemetry Conference in May. R.D. Shelton is assigned half-time to this task.

b) Orthogonal Multiplexing Systems Based on Easily-Generated Functions

Work will continue on evaluating the performance of orthomux systems based on simple waveforms. This work is especially important, since it has been determined that equipment simplicity is the only important criterion of comparison for certain channels. It is planned to present the key results at the IEEE International Convention. Tom Williams is assigned full time to this task. He has the assistance of an unpaid M.S. candidate.

c) Analog and Digital Simulation of Multiplex Systems

Work will continue on the simulations using the analog and digital computers. Measurements of crosstalk, noise performance, etc. will be made to verify the analytical calculations. Plans are being made to utilize the new hybrid computer when it becomes available. Three seniors are assigned to this task, each for one-quarter time, under the supervision of T. Williams and S. Riter. It is planned to present the key results at the IEEE Communications Conference in June.

d) Design of a Space Communications System for Advanced Manned Missions

A design based on conventional modulation and multiplexing systems has been completed. Work has just begun on an optimum all-digital PSK system. Dr. Bargainer will work one quarter time starting February 1 on the digital processing portion. Stephen Riter will work full time on the signal design and overall system performance analysis. He has the assistance of an unpaid M.S. candidate (a NASA engineer) and a senior who is building a test set for some of the required circuits. The conventional design will be presented at SWIEEECO in April.

e) Design of a Pictorial Transmission System for Advanced Manned Missions

Dr. Michaels will start February 1 to survey the state-of-the-art in picture transmission techniques to determine the optimum system for transmission of such information from deep-space. He will work one quarter time on the project.

C. Administrative Details

1. Personnel

a) Paid

Name	Title	Start Date	Finish Date	Rate
R.D. Shelton	Principal Investigator	6-1-66	6-1-67	½ time
H.S. Hayre	Professor	9-1-66	6-1-67	¼ time
E.L. Michaels	Professor	2-1-67	6-1-67	¼ time
J.D. Bargainer	Assistant Professor	2-1-67	6-1-67	¼ time
T. Williams	Research Associate	9-1-66	6-1-67	Full time
S. Riter	Research Associate	10-1-66	6-1-67	Full time
S. Wade	Technical Writer	6-1-66	6-1-67	2/3 time
L. Puigjaner	Graduate Assistant	6-1-66	9-15-66	½ time
P. Weinreb	Graduate Assistant	6-1-66	9-1-66	½ time
C. Osborn	Research Assistant	11-1-66	6-1-67	¼ time
S. Sloan	Teaching Fellow	10-1-66	6-1-67	¼ time
W. Trainor	Research Assistant	11-1-66	6-1-67	¼ time
M.L. Butler	Draftsman	6-1-66	9-1-66	½ time
E. Lozo	Draftsman	9-1-66	10-1-66	¼ time
J. Hennessy	Draftsman	10-1-66	6-1-67	¼ time
P. Stone	Typist	9-1-66	6-1-67	¼ time
W.L. Hon	Research Assistant	12-1-66	6-1-67	¼ time

b) Unpaid (Part time graduate students writing M.S. theses)

R. Van Cleave	Graduate Assistant	9-1-66	6-1-67	¼ time
B. Batson	Graduate Assistant	9-1-66	6-1-67	¼ time